SAFETY ENGINEERING OF ANTHROPOGENIC OBJECTS

COMBINATIONS OF ACTIONS FOR ACCIDENTAL DESIGN SITUATION: A REVIEW, ANALYSIS AND PROPOSITIONS

Viktor Tur  
Brest State Technical University  
ORCID: 0000-0001-6046-1974

Andrei Tur  
Brest State Technical University

Aliaksandr Lizahub  
Brest State Technical University

Stanislav Derechennik  
Brest State Technical University

Abstract

The paper discusses two main design strategies when checking for reliability and considers accidental action combinations according the various codes. If accidental actions can be identified, one of the possible design strategies is checking the “key element”. This strategy minimizes the possibility of local failure and subsequent progressive collapse. The combination of actions for accidental design situation for checking of the “key-element” resistance was proposed. In addition, the values of the combination factors for variable loads and partial factors for permanent loads in accordance with required reliability class RC for structural element and values of accidental loads was proposed. The second strategy is checking modified structural systems in accidental design situation from unidentified accidental actions. For this case, a comparison of several probabilistic models was performed, as well as a probabilistic assessment of the accidental action combinations according the various codes.

Key words: combinations of actions, accidental design situation, robustness, key element, modified structural systems.

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INTRODUCTION

Most of the structural codes ISO 2394 (2015); JCSS PMC (2001); EN 1991-1-7 (2006); EN 1990 (2009) provides a description of principles and application rules for the design of structural systems subjected to accidental action, including impact forces, actions due to internal explosions and due to local failure (Van Coile et al., 2019).

According to EN 1991-1-7 (2006) two groups of strategies are proposed in order to assess accidental design situations: those based on identified accidental actions and those based on limiting the extent of localized failure.

In the first group, it is proposed to design the structure to have sufficient minimum robustness, to prevent or reduce the effect of accidental action or to directly design the structural system to sustain the action.

The second group of strategies is based on limiting the extent of localized failure, either by increasing redundancy of structure or designing “key elements” to sustain notional accidental actions and applying some prescriptive rules like integrity or ductility.

Ellingwood et al. (2007) proposed a following formula to assess the probability of progressive collapse:

\[ P(C) = P(C|DH) \cdot P(D|H) \cdot P(H) \] (1)

where

- \( P(C) \) is the probability of progressive collapse;
- \( P(H) \) is the probability of the occurrence of a hazard \( H \);
- \( P(D|H) \) is the probability of local damage \( D \) as a result of a hazard \( H \);
- \( P(C|DH) \) – the probability of progressive collapse \( C \) of structural system as a result of local damage \( D \) caused by hazard \( H \).

In (Kokot, and Solomos, 2012) there is a good illustration of this expression (1) together with assigned appropriate terms (see Figure 1).

Considering the above Eq. (1) and Figure 1, the probability of progressive collapse can be minimized in three ways, namely by: controlling abnormal events (term \( P(H) \), controlling local element behavior (term \( P(D|H) \)) and/or controlling global system behavior (conventional probability \( P(C|DH) \)).
It is worth nothing in (Kokot, and Solomos, 2012), that controlling abnormal events by structural engineers is normally very difficult, practically impossible. However, engineer can influence the local and global system behavior, i.e. probabilities $P(D|H)$ and $P(C|DH)$.

Conditional probabilities presented in Eq. (1) can be obtained by a probabilistic risk analysis (PRA), in which it is possible to model uncertainties, study their propagation and the effect on the required performance of the structural systems (with damaged elements). This approach is called structural reliability analysis and failure (collapse in the case in question) is considered achieved when demand $E$ (i.e. the effects generated by the actions) exceeds collapse resistance $R$. In general case, the probability of failure is equal to:

$$p_f = \int F_R(x)f_E(x)dx$$

(2)

where $F_R(x)$ is the CDF (cumulative distribution function) of resistance $R$ and $f_E(x)$ is the PDF (probability density function) of $E$ (effect of actions).

The probability of disproportionate collapse can be defined according to EN 1990 (2009) as follows:

$$p_f = \text{Prob}[E \leq R] \text{ or } p_f = \Phi(-\beta)$$

(3)

where $\beta$ is the reliability index for structural system and $\Phi(\bullet)$ is a normal standard distribution function.

For a correct assessment of disproportionate collapse risk, it may be necessary to consider the presence of multiple hazard events and the initial stage of damage. In this case, Eq. (1) can be generalized as illustrated in the following equation (valid for independent event only):

$$P(C) = P(C|DH)\cdot P(D|H)\cdot P(H)$$

Figure 1. Terms in the context of progressive collapse (from Kokot, and Solomos, 2012).
\[ P(C) = P(C|DH)P(D|H)\lambda_H \]

where \( \lambda_H \) can substitute \( P(H) \) if occurrence probability is less than \( 10^{-2}/\text{year} \). Values of \( \lambda_H \) are reported in (Ellingwood et al., 2007).

As was shown in (Ellingwood, 2005), if a performance based design approach is adopted, an acceptable value of risk tolerance has to be defined. In the case of a disproportional collapse, which main consequence is the loss of human life, decision-makers can assume that the performance objective of safeguarding human life is achieved if the following relationship is verified:

\[ P(C) \leq p_{\text{tag,h}} \]

where \( p_{\text{tag,h}} \) is the risk threshold defined as “de minimis” which in general case assumed values ranging from \( 10^{-5}/\text{year} \) and \( 10^{-7}/\text{year} \). More detailed discussion presents in our publication (Tur et al., 2019).

Moreover, in particular case in which so called alternative load path method (ALP-method (Arup, 2011; Ellingwood et al., 2007)) is used in design phase, the collapse probability becomes \( P(C|DH) \), which in turn has to respect the following equation according to (Ellingwood et al., 2007):

\[ P(C|DH) < \frac{p_{\text{tag,h}}}{\lambda_H} \]

Therefore, assuming \( \lambda_H \) equal to \( 10^{-6}/\text{year}..10^{-5}/\text{year} \), the performance based target probability established by condition (6) requires that the conditional probability of collapse for the modified structural system be in the order of \( 10^{-2}/\text{year}..10^{-1}/\text{year} \).

Consequently, as shown in (Ellingwood, 2005), the reference reliability index \( \beta_0 \) for the limit collapse state of conditioned by the occurrence of the damage will be in order of 1.5. That is significantly lower than that assumed for ultimate limit state of new buildings for
residential and office use in case of ordinary actions (i.e. $\beta_{aug} = 3.8$, which corresponds to reference probability for structural system collapse of the order of $\sim 10^{-4}$).

### 1.1 Combinations of the actions according to various codes

According to (Gulvanessian, 2020; Arup, 2011), a reliability based approach can be applied to determine reasonable loading combinations for accidental design situation. The actions to be combined reflect the small probability of a joint occurrence of the accidental action and design values of imposed (or live), snow, wind loads.

Hazard events, and mainly, malicious attack are a rare events and many of them suppressed early.

Focusing on the mechanical actions, these are traditionally subdivided into permanent actions and imposed (variable) action according to (CIB, 1989). Their variability with time is an aspect of particular relevance for checking of the structural system in accidental design situation. As was shown in (Gulvanessian, 2020), in partial factor design method (PFM) for normal conditions, the load variability is considered by a characteristic or design load with a low probability of being exceeded during the service life of the structure. This ensures that the building structure are designed both safety and economically, as in setting the design requirements a balance has been sought between the cost of premature failures and the cost of additional safety investment (see ISO 2394 (2015)).

Figure 2 shows that reliability indices (failure probabilities) are influenced by the efficiency of safety investments and consequences of failure (ISO 2394, 2015). The optimal reliability index $\beta^*$ can be obtained by minimizing the sum of investments in safety measures and the accompanying capitalized risk.

**Figure 2.** Principles of cost minimization, reliability optimum $\beta^*$ and reliability minimum $\beta_{aug,h}$ according to ISO2394 (2015).
The target reliability indices derived on the basis of economic optimization might not be acceptable with regard to requirements concerning human safety, as it is stated in ISO2394 (2015). These reliability indices are denoted as $\beta_{tag,h}$.

It is clear, the day-to-day probability of occurrence of such high (design) load value is low, just as for the day-to-day probability of occurrence of a hazard event (accidental event). Simultaneously taking into account both events would result in very onerous design requirement for accidental design situation (in case of modified structural system robustness checking).

Hence, the reduced partial safety and combination factors in EN 1991-1-7 (2006), ASCE 7 (2005) and other codes (BS 6399, 1996; GSA, 2003; UFC 4-023-03, 2005) (see Table 1) lesser the required load under consideration for structural design in accidental design situation compared to normal design situation.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Load combinations</th>
<th>Combination number for Table 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS 6399 (1996)</td>
<td>$D + L/3 + W/3$</td>
<td>(1)</td>
</tr>
<tr>
<td>EN 1991-1-7 (2006)</td>
<td>$\sum G + P + A_d + \psi_1 Q_{k,1} + \sum \psi_2 Q_{k,1}$</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>$\sum G + P + A_d + \psi_2 Q_{k,1} + \sum \psi_2 Q_{k,1}$</td>
<td>(3)</td>
</tr>
<tr>
<td>ASCE 7 (2005)</td>
<td>$(0.9D \text{ or } 1.2D) + (0.5L \text{ or } 0.2S) + 0.2W_n$ – alternate load path method</td>
<td>(4) or (5)</td>
</tr>
<tr>
<td></td>
<td>$1.2D + A_k + (0.5L \text{ or } 0.2S)$ – specific local resistance method</td>
<td>(6)</td>
</tr>
<tr>
<td>GSA (2003)</td>
<td>$2(D + 0.25L)$ – static analysis</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D + 0.25L$ – dynamic analysis</td>
<td></td>
</tr>
<tr>
<td>UFC 4-023-03 (2005)</td>
<td>$(0.9D \text{ or } 1.2D) + (0.5L \text{ or } 0.2S) + 0.2W_n$ – nonlinear dynamic analysis</td>
<td>(7) or (8)</td>
</tr>
<tr>
<td></td>
<td>$2[(0.9D \text{ or } 1.2D) + (0.5L \text{ or } 0.2S) + 0.2W_n]$ – static analysis</td>
<td></td>
</tr>
</tbody>
</table>


**Table 1. Accidental action combinations according the various codes.**

### 1.2 Load combinations for accidental design situation (key-elements checking)

As shown above (Gulvanessian, 2020), in general case, hazard events can be classified in two major types: unintentional but identified (Natural and Accidental) hazards and malicious attacks. According to (Gulvanessian, 2020), the distinct, nature of two types of hazard implies that the hazard associated uncertainties, severity and frequency of occurrences are significantly different. For unintentional hazards such as earthquake, wind, scour, vessel collision, random stochastic models are typically used to represent the hazard intensity and occurrence. However, for purposely plotted malicious destruction such as explosions and
intentional collisions and purposely made accidents (criminal and terrorist attacks), the 
ordinary random stochastic model is not longer valid.

According to EN 1990 (2009) and EN 1991-1-7 (2006) the general format of effects of 
actions for the accidental design situations is analogous to the general format for STR/GEO 
ultimate limit states. Here, the loading action is the accidental action, and the most general 
expression of the design value of the effects of actions is the following:

$$E_d = E\{G_{k,j}; P_i; A_d; (\psi_{1,1} \text{ or } \psi_{2,1})Q_{k,i}; \psi_{2,i}Q_{k,i}\} \ (7)$$

which can also be expressed as:

$$\sum_{j=1}^{G_{k,j}} (\psi_{1,1} \text{ or } \psi_{2,1})Q_{k,i} + (A_d)Q_{k,i} \ (8)$$

According to background to EN 1990 (2009), this combination considers that:
- accidents are unintended events such as explosions, fire or vehicular impact, which are of 
very short duration and have a low probability of occurrence;
- a certain amount of damage is generally acceptable in the event of an accident;
- accidents generally occur when structures are in use.

Hence, to provide a realistic accidental combination, accidental actions are applied 
directly, with the frequent and quasi-permanent combination values used for the main (if any) 
and other variable actions respectively (see cl. 6.4.3.3 (3) EN 1990 (2009)).

Regarding the representative value (frequent and quasi-permanent) of a possible main 
variable action, EN 1990 (2009) states that discretion is left to national authorities for the 
reason that all accidental situations or events cannot be similarly treated. When the main 
variable action is not obvious, each variable action should be considered in turn as the main 
action.

The combination for accidental design situation either involve an explicit design value of 
accidental action $A_d$ (e.g. impact) or refer to a situation after an accidental event ($A_d = 0$).

The partial factors for actions for ultimate limit states in the accidental design situations are 
normally taken equal to 1.0, in general, not only are the reliability elements for actions 
modified for the partial factors for resistances.
Based on results of our own investigations (Tur, and Markovskij, 2009), we proposed to use for checking of the “key-element” resistance the following combination of actions for accidental design situation (combination comprises accidental action $A_d \neq 0$):

$$E_{d,A} = \sum_{j=1}^{n} \left( \gamma_{GA,j} \right) G_{k,j} + A_d + \psi_{A,1} Q_{k,1}$$

(9)

where $G_{k,j}$ is the characteristic value of a permanent action “$j$”;

$Q_{k,1}$ is the characteristic value of the leading variable action;

$A_d$ is the design value of the accidental action;

$\gamma_{GA,j}$ is the combination factor applied to a permanent action “$j$”;

$\psi_{A,1}$ is the combination factor applied to the leading variable action according to Table 2.

In Table 2, we relate values of the combination factors $\psi_A$ with required reliability class $RC$ for structural element and factor $k$, which is determined as ratio:

$$k = \frac{A_d}{E_k}$$

(10)

with

$$E_k = \sum_{j=1}^{n} G_{k,j} + Q_{k,1} + \sum_{i>1} Q_{k,i}$$

(11)

where $Q_{k,i}$ is the characteristic value of the accompanying variable actions.
Reliability Class | $k_A$ | Imposed $(Q)_A, \psi_{AQ}$ | Wind $(W)_A, \psi_{AW}$ | Snow $(S)_A, \psi_{AS}$ | Permanent $(G), \psi_{GA}$
--- | --- | --- | --- | --- | ---
RC2 | 1.0 | 0.8 | 0.8 | 0.7 |
| 1.5 | 0.6 | 0.6 | 0.55 |
| 2.0 | 0.5 | 0.5 | 0.4 |
| 2.5 | 0.35 | 0.4 | 0.3 |
| 3.0 | 0.2 | 0.3 | 0.2 |
| 3.5 | 0.1 | 0.2 | 0.1 |
| 4.0 | 0.05 | 0.15 | 0.05 |
| | | | | 1.0 |
RC3 | 1.0 | 1.0 | 1.05 | 1.0 |
| 1.5 | 0.9 | 0.95 | 0.85 |
| 2.0 | 0.8 | 0.8 | 0.7 |
| 2.5 | 0.7 | 0.7 | 0.6 |
| 3.0 | 0.55 | 0.6 | 0.5 |
| 3.5 | 0.45 | 0.5 | 0.4 |
| 4.0 | 0.4 | 0.45 | 0.35 |
| | | | | 1.05 |

Table 2. Combination factors $\psi_A$ and partial factors $\gamma_{GA}$ for checking of the “key-element” resistance.

1.3 Combinations of actions for checking modified (damaged) structural systems in accidental design situation

It should be noted that the code EN 1991-1-7 (2006) propose to apply the same combination of action to check the resistance of “key-elements” under the accidental action ($A_d \neq 0$) and to check the robustness of the modified structural system after removing the damaged element, taking $A_d = 0$. In this case, the values of the combination coefficient $\psi_i$ remain unchanged. On the other hand, various codes (BS 6399, 1996; ASCE 7, 2005; GSA, 2003; UFC 4-023-03, 2005) do not consider accidental combinations that include accidental loads $A_d$ (see Table 1). From our point of view, this approach is not entirely correct.

Firstly, when calibrating partial (combination) factors in combination (8) which are used to design of “key-elements” under accidental load $A_d$, load models are formulated for reference period equal to in most cases service life of building.

For assessment of the robustness of the modified structural system after removing the damaged element, load models should be formulated for the other reference period $T_{ref}$ differs from service life (it may be evacuation time, a period of destroying, dismounting or reconstruction).
As it was shown in our publications (Tur, and Markovskij, 2009) these reference period be from 1 day to 3 months.

Secondly, in the first case (key-elements design) calibration of the partial factors in accidental combination (8) performs with the usage of the state function including element resistance function in closed form. However, in the second case (system robustness assessment) to formulate resistance function for the damaged structural system in closed form is practically impossible.

As shown prior analysis of the expressions listed in Table 1, various structural codes use different load combinations for accidental design situation. In the general case, an accidental load combination includes permanent, climatic (snow and wind actions) and imposed (live) load. Prior analysis of the expressions from Table 1 shown that values of partial coefficients applied with the characteristic loads are sufficiently different. Therefore, the total value of accidental load corresponds to various quantiles of CDF (for the total load combination $K_e(Q+G)$) and, consequently provides a different reliability level for the same characteristic values of actions.

In an accidental design situation, when structural engineer considers malicious terrorist and criminal attacks (unidentified hazards), accounting of the climatic (snow and wind) actions jointly with the imposed load in accidental load combinations, makes no sense.

The first, at the stage of the attack planning, it is very difficult and practically impossible, to foresee real point-in-time when the maximum value of the climatic actions will appear simultaneously with the design value of the imposed load.

If the wind action can have a significant influence on the high-rise building structural behavior in an accidental design situation, snow load influence is insignificant with the RC-buildings mainly (the maximum part of the snow load in total gravity load in near 15% only).

Therefore, the total combination that includes an imposed load for the assessment of the modified structural system robustness in an accidental design situation is decisive. In the general case, we should consider two types of the imposed load when accidental load combination is developed:

1) only sustained imposed load (as more realistic load value for day-to-day exploitation);
2) total value of the imposed load (sustained plus intermittent parts) for extraordinary event.

It should be noted that when the probabilistic modelling applying accidental action shall be considered as an impulse at-any-time-point. Such impulse has a very high intensity and a short period of action in comparison with permanent and sustained imposed (variable) loads.
As the occurrence of the intermittent (transient) imposed load is by its conceptualization rare, it generally does not need to be taken into account simultaneously with accident (hazard) (Van Coile et al., 2019). While this can be considered sufficient for the general floor area of most buildings (e.g. offices, residential buildings), care should be taken whenever the imposed (live) load profile of building has specific occurrence patterns or particular likelihood of overcrowding (e.g. sports stadia), or when considering buildings with a high reliability requirements (e.g. high-rise buildings). The possible overcrowding near emergency exits during the evacuation process may need to be considered, although this is (partially) compensated by the necessary absence of furniture in those areas. This aspect is not considered further here, and the models specified below are not developed to apply to evacuation routes (Van Coile et al., 2019).

Wherein intermittent part of the imposed load not considered in the load model because of this part of the imposed load describe as an impulse too. It is very low probability of the simultaneous appearance of the two impulses of the accidental and intermittent loads. The sustained part of the imposed load describes as arbitrary-point-in-time load.

2.1 Imposed load modelling

With reference to the discussion in (Van Coile et al., 2019; CIB, 1989) about the two different parts of the imposed (live) load it may be convenient to divide the live load into two components:

- sustained load;
- intermittent load.

The sustained load contains the weight of furniture and heavy equipment. The short term fluctuations have been smoothed and thus the real load, which is shown in Figure 3a as a function of time, is simplified to give the load model shown in Figure 3b. The load magnitude according to the model is supposed to represent approximately the time average of the real fluctuations will be included in the uncertainties of the sustained load.

Furthermore, the sustained load also includes the weight of persons who a normally present. This load is here regarded as constant in time (between the changes of occupancy) in the same way as the fluctuating part of the weight of furniture and heavy equipment. This is justified in those case when the sustained load which is caused by the weight of persons normally present is small.
The situation may be quite different for lecture room in schools and other similar types of buildings (conference buildings) where the weight of persons normally present is a large part of the total live load. For these types of room, the load caused by persons normally present may be treated specially. It may, for example, be modelled according to Figure 3c.

In the load model the intermittent load is assumed to represent all kinds of imposed load which are not covered by the sustained load. Thus intermittent load may have many different sources, such as the examples mentioned below. Examples of such situations could be:

a) the gathering of people during special planned events, such as parties. During such events people tends to cluster into group;

b) the crowing of people under emergency type situations, such as in an exit hall or on a fire escape;
c) the piling up of furniture in one area while surrounded areas are being remodelled. In such case, the total load caused by furniture may be unchanged but it is concentrated in a smaller floor area.

In case (a) and (b) the relative duration of intermittent load is fairly small, in case (c) it may be greater. The occurrence of the intermittent live load can be illustrated according to Figure 3d.

The combined sustained and intermittent live load is shown in Figure 3e.

During the design of a structure different load values are importance for different design situations. The maximum total live load occurring during a previously selected reference period is in most cases decisive for safety problems. During some period of time or at some point in time the maximum sustained load will occur as indicated in Figure 3b.

The intermittent load has normally one particular occurrence producing its maximum magnitude (see Figure 3d). The maximum total load for the combined process (see Figure 3e) might occur when the sustained load is at its maximum, when the intermittent load is at its maximum or when neither of them is at its respective maximum.

### 2.2 The maximum of the sustained load

If the maximum of the sustained load is of interest, it is normally sufficient to consider the marginal statistical distribution $F_s(x)$ (the index “s” means sustained), of the sequence of $n$ independent loads (JCSS PMC, 2001; CIB, 1989). The probability distribution function for the maximum load is given by:

$$F_{max,s}(x) = \sum_{n=1}^{\infty} [F_s(x)]^n P(N = n)$$

where the number $N$ load events can be deterministic or determined by some statistical distribution.

In the case of a deterministic value of $N, N = n$, the widely used Ferry Borges load model is obtained.
If the time between load changes is exponentially distributed then the number of load changes is Poisson distributed. With this assumption Eq. (12) yields

\[
F_{\text{max},s}(x) = \exp\left[-vT \left(1 - F_s(x)\right)\right]
\]

where \( T \) is an appropriate reference time, for example, the anticipated life time of the building;

\( v \) is the occurrence rate of sustained load changes.

Thus \( vT \) is the mean of number of occupancy changes.

For the upper tail of \( F_{\text{max},s}(x) \), i.e. if the value of \( F_{\text{max},s}(x) \) is greater than about 0.8, Eq. (13) and Eq. (14) give nearly the same values. A Poisson distribution which emphasizes very short durations may not be appropriate for application.

A similar expression to Eq. (14) but more complicated, can be derived for Gamma distributed number of load events. The result can be obtained in a close form only for Erlang distribution.

A common procedure is to evaluate Eq. (14) at two different cumulative values in the upper tail and match a Type I extreme value distribution of these values.

### 2.3 The maximum of the intermittent load

In general case the maximum load which occur in a building is a combination of sustained loads and intermittent loads. As stated in (CIB, 1989), in most cases it is reasonable to assume that the sustained load and the intermittent load are mutually independent. However, a dependence may exist in special cases.

The maximum of the intermittent load during one occupancy, the duration of which is assumed to be Erlang distributed (Gamma distributed with an integer value of the shape parameter), is given by (CIB, 1989):
\begin{equation}
F_{(\text{max},p),1}(x) = \frac{x^k}{(v + \rho(1 - F_\rho(x)))^k} 
\end{equation}

where $F_\rho(x)$ is the probability distribution function of the intermittent load; 
\( \rho \) is the occurrence rate of intermittent loads; 
\( k \) is the shape factor in the Erlang distribution ($k = 1, 2, \ldots$).

For example, $k = 1$ in the special case when the time intervals between the loads are exponentially distributed.

The maximum total load during one occupancy is obtained from the convolution integral:

\begin{equation}
F_{\text{max} (s+p),1}(x) = \int_0^x F_{\text{max} (s+p),1}(x-z)f_s(z)dz
\end{equation}

where $f_s(z)$ is the probability density function for the sustained load during one occupancy.

**2.4 The total maximum imposed load**

The total maximum imposed load during the entire reference period can be obtained, see (CIB, 1989), by considering the simultaneous distribution of completed durations. Unfortunately, this does not lead to a closed expression. The total maximum load during the reference period $T$ can then be expressed as:

\begin{equation}
F_{\text{max} (s+p)}(x) \approx \exp\left[-\nu T\left(1 - F_{\text{max} (s+p),1}(x)\right)\right]
\end{equation}

where $T$ and $\nu$ are the same as for Eq. (14).

**2.5 Design total accidental action modelling**

Different permanent load and imposed load models have been proposed for the structural systems checking in an accidental design situation.
It should be pointed that these studies make limited explicit reference to the issue of time variability of the load. Most stating (directly or indirectly) that their load models correspond with arbitrary-point-in-time (APIT) permanent and imposed loads, e.g. (Van Coile et al., 2019). The study by (Ellingwood, and Culver, 1977; Ellingwood, 2005) is a notable exception, going to some depth in explaining underlying process of loading variability.

The overview of applied permanent load models is given in Table 3 (where $\mu$ is the mean value, $V$ is the coefficient of variation (CoV), and $G_{\text{nom}}$ is the nominal permanent load). Similarly, Table 4 gives an overview of imposed load models, showing a larger variation in models. For the theoretical imposed load model, reference is made to (Van Coile et al., 2019; Ellingwood, and Culver, 1977; CIB, 1989).

<table>
<thead>
<tr>
<th>Study</th>
<th>Distribution</th>
<th>$\mu/G_{k}$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guo et al., 2013; Iqbal, and Harichandran, 2010; Hamilton, 2010; Ellingwood, 2005</td>
<td>Normal</td>
<td>1.05</td>
<td>0.10</td>
</tr>
<tr>
<td>Devaney, 2015; Van Coile et al., 2014; Holicky, and Schleich, 2005</td>
<td>Normal</td>
<td>1.00</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Table 3. Permanent load models (Van Coile et al., 2019).**

<table>
<thead>
<tr>
<th>Study</th>
<th>Distribution</th>
<th>$\mu/Q_{k}$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guo et al., 2013; Iqbal, and Harichandran, 2010</td>
<td>Gamma</td>
<td>0.24</td>
<td>0.8..0.6</td>
</tr>
<tr>
<td>Hamilton, 2010; Ellingwood, 2005</td>
<td>Gamma</td>
<td>0.24 to 0.50</td>
<td>0.6</td>
</tr>
<tr>
<td>Devaney, 2015; Van Coile et al., 2014</td>
<td>Gumbel</td>
<td>0.6</td>
<td>0.35</td>
</tr>
<tr>
<td>Holicky, and Schleich, 2005</td>
<td>Gumbel</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Hosser et al., 2008</td>
<td>Gumbel</td>
<td>0.52</td>
<td>0.5</td>
</tr>
<tr>
<td>Van Coile et al., 2019; Gernay et al., 2019; Holicky, and Schleich, 2005</td>
<td>Gumbel</td>
<td>0.2</td>
<td>1.1</td>
</tr>
</tbody>
</table>

**Table 4. Imposed load models (Van Coile et al., 2019).**
The widely cited paper by (Ellingwood, and Culver, 1977; Ellingwood, 2005) does not specify a clear formulation for the total load model.

In (Van Coile et al., 2019) on the other hand, the total load model of Eq. (18) is used, where $K_E$ is the model uncertainty for the load effect:

$$w = K_E (G + Q)$$

To assess the effect of the different load models, the total load formulations are compared. To make a direct comparison possible, the load ratio $\chi$ and total characteristic (nominal) load $P_k$ are defined through Eq. (19), where the characteristic (nominal) values $Q_k$ and $G_k$ when using the Eurocode methodology:

$$\chi = \frac{Q_k}{G_k + Q_k} = \frac{Q_k}{P_k}$$

and

$$\xi = \frac{w}{P_k} = K_E \left((1-\chi) \cdot g_k + \chi \cdot q_k\right)$$

The variation in $G$ and $Q$ is thus taken into account through the stochastic variables $g$ and $q$ with $\mu$ and $V$ as listed in Tables 3, 4.

Considered the background discussion in (Van Coile et al., 2019) (see Tables 3, 4), the permanent load can be described by a normal distribution, with a mean value slightly exceeding its nominal value, and a CoV which can be evaluated according to (JCSS PMC, 2001).

According to (Van Coile et al., 2019), the permanent load effect $G$ is recommended to be described by a normal distribution, with mean equal to the nominal permanent load effect $G_{nom}$ and CoV=0.10.

It should be noted that researchers (Van Coile et al., 2019; Guo et al., 2013; Guo, and Jeffers, 2015; Iqbal, and Harichandran, 2010; Hamilton, 2010; Van Coile et al., 2014; Holicky, and Schleich, 2001; Hosser et al., 2008; Gernay et al., 2019; Holicky, and Schleich,
2005; Devaney, 2015) use different statistical descriptions of the basic variables in the total load model for accidental design situation.

Let’s consider the most commonly used models of the basic variables for accidental load combination and determine the reliability levels that provide accidental load combinations adopted in various codes (see Tables 5-7). This comparison is performed based on a cumulative density function (CDF) for total accidental load proposed in original sources (Van Coile et al., 2019; Holicky, and Schleich, 2005; JCSS PMC, 2001).

### 2.6 Load model by Van Coil et al. (2019)

Table 5 gives the stochastic parameters of the basic variables of proposed model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Load component</th>
<th>$V$</th>
<th>$\mu$/nom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_E$</td>
<td>LogNormal</td>
<td>Model uncertainty</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$G$</td>
<td>Normal</td>
<td>Permanent load</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$Q$</td>
<td>Gamma</td>
<td>Imposed load</td>
<td>0.95/0.6*</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Note: * – for small/large loaded area.

**Table 5. Stochastic parameters of the basic variables of model by (Van Coile et al., 2019).**

### 2.7 Load model according to Eurocode (by Holicky and Schleich (2005))

The second family of APIT imposed load models in Table 6 consider a 5-years Gumbel distribution. The references listed refer to EN 1991-1-7 (2006) background documents and the 2010 review of stochastic models by (Holycky, and Sykora, 2010). Considering, the 5-years Gumbel distribution specified in (Holycky, and Sykora, 2010) related to office buildings designed in accordance with EN 1990 (2009); EN 1991-1-7 (2006) recommended characteristic imposed load of 2.0 to 3.0 kPa. The factor 0.2 listed in Table 6 then corresponds with a mean value of 0.4..0.6 kPa for sustained imposed load, which is in agreement both with the values specified above with reference to (Van Coile et al., 2019) and (Guo et al., 2013;
Guo, and Jeffers, 2015; Iqbal, and Harichandran, 2010; Hamilton, 2010), as well as with the mean value of 0.5 listed by Holicky and Sykora (2010), referencing (Van Coile et al., 2019).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Load component</th>
<th>$V$</th>
<th>$\mu$/nom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_L$</td>
<td>Normal</td>
<td>Model uncertainty</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$G$</td>
<td>Normal</td>
<td>Permanent load</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$Q$</td>
<td>Gumbel</td>
<td>Imposed load</td>
<td>1.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 6. Stochastic parameters of the load model according to Eurocode (by Holicky and Schleich (2005)).

2.8 Load model according to JCSS PMC (2001)

The Probabilistic Model Code specifies a Gamma distribution for the instantaneous sustained load, as noted also in JCSS PMC (2001). For different occupancies, distribution parameters are tabulated in Table 7, according to JCSS PMC (2001).

<table>
<thead>
<tr>
<th>Category</th>
<th>$A_0$</th>
<th>$\mu_q$</th>
<th>$\sigma_V$</th>
<th>$\sigma_U$</th>
<th>$1/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office</td>
<td>20</td>
<td>0.5</td>
<td>0.3</td>
<td>0.6</td>
<td>5</td>
</tr>
<tr>
<td>Residence</td>
<td>20</td>
<td>0.3</td>
<td>0.15</td>
<td>0.3</td>
<td>7</td>
</tr>
<tr>
<td>Classroom</td>
<td>100</td>
<td>0.6</td>
<td>0.15</td>
<td>0.4</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 7. Parameters of the imposed sustained load in accordance with loading areas by JCSS PMC (2001).

The standard deviation of this imposed load model is calculated by Eq. (21), with $\sigma_V$ the standard deviation of the overall load intensity, $\sigma_U$ the standard deviation associated with the spatial variation of the load, $A_0$ an occupancy-specific reference area, $A$ the loaded area and $k$ an influence factor (commonly between 1 and 2.4; further taken as 2.2 for agreement with (Ellingwood, and Culver, 1977).
\[ \sigma^2 = \sigma_{\nu}^2 + \sigma_{\nu}^2 \cdot k \cdot \min \left\{ \frac{A_b}{A} ; 1 \right\} \] (21)

The JCSS PMC (2001) further notes that one of the underlying assumptions for the equivalent uniformly distributed load model is a linear structural response.

The assumption of linearity can be omitted by considering the spatial variability of the load explicity. The latter is however considered too demanding for practical feasibility. Nonlinear behavior could be considered as part of the model uncertainty \( K_E \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Load component</th>
<th>( V )</th>
<th>( \mu/\text{nom} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_E )</td>
<td>Normal</td>
<td>Model uncertainty</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>( G )</td>
<td>Normal</td>
<td>Permanent load</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>( Q )</td>
<td>Gamma</td>
<td>Sustained load</td>
<td>depends on ( A ) and ( k )</td>
<td>depends on category of loading area</td>
</tr>
</tbody>
</table>

Table 8. Stochastic parameters of the load model according to JCSS PMC (2001).

3 Probability analysis of combinations for checking modified (damaged) structural systems in accidental design situation

All models (see Tables 5-8) have been evaluated using \( 10^8 \) crude Monte Carlo Simulations (MCS), for load ratio \( \chi \), applying the distribution model according to Tables 5-8.

Example of obtained cumulative density functions (CDF) for the total characteristic load factor \( \xi \) according to the reviewed in Table 1 load models with different CoV values are given in Figure 4.
Table 9 shows the exceedance probabilities corresponding to the total accidental load combinations included in various codes (see Table 1), obtained from the CDF for the various reviewed load models and various category of occupancy.

As can be seen from Figure 4 values of the total accidental load calculated based on the various load combinations according to different codes (see Table 9) and corresponding exceedance probability of CDF of this total action (see Eq. (18)), varies in very wide interval (from 7% to 52%).

Moreover, Table 9 shows that for the various category of occupancy accidental load combination according one code gives various values of quantilies of CDF and reliability level.
The total characteristic load factor $\xi$:

<table>
<thead>
<tr>
<th>Category</th>
<th>Probability model</th>
<th>Codes (Load combinations for progressive collapse analysis according to Table 1)</th>
<th>BS</th>
<th>EC</th>
<th>ASCE 7-05</th>
<th>GSA</th>
<th>UFC 4-023-03</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The total characteristic load factor $\xi$:</strong></td>
<td></td>
<td></td>
<td>0.80</td>
<td>0.85</td>
<td>0.79</td>
<td>0.78</td>
<td>0.99</td>
</tr>
<tr>
<td>Office $A = 50 \text{ m}^2; k = 1.4$</td>
<td>Eurocode by (Holicky, and Schleich, 2005)</td>
<td></td>
<td>0.40</td>
<td>0.29</td>
<td>0.42</td>
<td>0.44</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>JCSS PMC, 2001</td>
<td></td>
<td>0.37</td>
<td>0.26</td>
<td>0.39</td>
<td>0.42</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>Van Coile et al., 2019</td>
<td></td>
<td>0.39</td>
<td>0.27</td>
<td>0.41</td>
<td>0.44</td>
<td>0.08</td>
</tr>
<tr>
<td>Residence $A = 50 \text{ m}^2; k = 1.4$</td>
<td>Eurocode (Holicky, and Schleich, 2005)</td>
<td></td>
<td>0.43</td>
<td>0.35</td>
<td>0.44</td>
<td>0.52</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>JCSS PMC, 2001</td>
<td></td>
<td>0.40</td>
<td>0.32</td>
<td>0.42</td>
<td>0.50</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>Van Coile et al., 2019</td>
<td></td>
<td>0.42</td>
<td>0.34</td>
<td>0.44</td>
<td>0.52</td>
<td>0.10</td>
</tr>
<tr>
<td>Classroom $A = 100 \text{ m}^2; k = 1.4$</td>
<td>Eurocode (Holicky, and Schleich, 2005)</td>
<td></td>
<td>0.40</td>
<td>0.29</td>
<td>0.42</td>
<td>0.44</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>JCSS PMC, 2001</td>
<td></td>
<td>0.39</td>
<td>0.28</td>
<td>0.42</td>
<td>0.44</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>Van Coile et al., 2019</td>
<td></td>
<td>0.39</td>
<td>0.27</td>
<td>0.41</td>
<td>0.44</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 9. The exceedance probabilities corresponding to the total accidental load combinations included in various codes.

CONCLUSION

Proposed combination of actions for accidental design situation Eq. (9) gives resistance of the “key-element” in accordance with required reliability class. It was found that the application of the $\psi_A$ coefficients to the accompanying (non-dominant) loads does not lead to a change in the reliability indices. Because of the significant accidental action $A$, the influence of all other loads decreases. Only the leading variable action have noticeable influence on the reliability indices. In this regard in Eq. (9), only dominant variable loads are taken into account in combination of actions for accidental design situation for the “key” system elements. For non-dominant variable action $\psi_{A,i,2} = 0$.

The calibrated values of the combination factors $\psi_{A,i,1}$ for the key elements (see Table 2) depend on the required reliability class of structural elements and the factor $k$, which is
defined as the ratio of the design value of the effect from the accidental action $A_d$ on the element and the effect $E_k$ from the total characteristic load on this element. Depending on the magnitude of the accidental action, the values of the combination factors $\psi_{A,i,1}$ are for the imposed load from 0.05 to 1.0, for the wind load from 0.15 to 1.05, for the snow load from 0.05 to 1.0.

In situation of checking modified (damaged) structural systems in accidental design situation, it is very difficult to answer on question: What accidental combination is right, complete and reliable?

In general case, we have an uncertain solution because the low value of the CDF percentile can be compensated by higher values of the global safety factors (for example, according to EN 1990 (2009)) for non-linear resistance model. Conclusion about reliability of the analyzed structural system can be made based on the comparison of the obtained by calculation ($p_f$) and target ($p_{tag}$) values of the failure probability only. The probability of failure for structural system can be obtained based on state function consists of, in general case, the action model and resistance model. Even if we assuming the deterministic value can be obtained only in the case known resistance function for the modified structural system. This resistance function should be formulated for the every analyzed modified structural system based on non-linear calculation.

REFERENCES


*Symposium 2019: Concrete-Innovations in Materials, Design and Structures* (pp. 2126-2133).